Woodrow Wilson Seminar: Proficient but not College Ready: Bridging the Gap

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Slide 1 The issue that is before us – what is needed for college success - is an important one that is not enough treated by policy makers who often operate as if a particular score on the SAT or ACT is sufficient for success in college. In mathematics, these college entrance examination scores are particularly seductive because there is a score that is called mathematics, unlike, let's say, science or social studies, and a single number is such a simple barometer. Some colleges operate as if algebraic skill is the only indicator of success, another seductive barometer because skill is the easiest thing to assess.

Slide 2 My remarks today consist of five sections: transitions; linearity; prerequisites for success; national data; and the question whether the dream is possible.

Slide 3 1. Transitions Before the Transition to College

I would like to start from the ideas in the talk whose script turned into an article that was published ten years ago and sent to you last week. That talk argues that, from a content perspective, the transition into algebra and high school level mathematics plays a role that is parallel to the transition into calculus and college level mathematics. Specifically:

Slide 4 Algebra and calculus are both:

- fixtures of the curriculum; the first course at a new level; a sign of arrival
- prerequisites to a great deal of future work
- filters
- normally offered at a variety of levels of difficulty

- normally taken one year earlier by the best students.

Slide 5 Other common aspects are that:

- some people think that the basic ideas can be learned much earlier than one year before
- hurdles are in place to discourage early work

I could add here that readiness for calculus and readiness for algebra also have much in common. Early work in each subject is not necessarily harmful, but can be beneficial. In fact, a student today who comes into the first algebra course never having dealt with variables as generalizations or patterns, not having graphed points on a 2-dimensional grid, never having solved more than simple equations, and not having done algebraic simplifications is at a major disadvantage. It is the same with a student who comes into the first calculus course without having some introduction to calculus.

I would like to start, however, not with these specific content prerequisites for college success, but with the more general process ideas as noted in the David Conley essay, College Readiness. Therein is very fine detail of the differences between the expectations of high school and those of college. I would like to extend this idea down the grades because there are implications for our discussions. The transition to college is not the first transition a good student traverses in schooling. It is the fourth in a line of transitions I believe a person must make to be a successful college student.

Slide 6 The first transition comes upon going to school. Before going to school, the learning of the young child is informal, typically unplanned in advance, based on what comes up in life. It is also typically one on one, parent to child. The first-grader needs to adapt to a scheduled learning with some expectations of performance even if there are no tests. This learning takes place along with 18-30 other students and without the parent present. No wonder there is so much written about the first day of school!

How does the parent or guardian prepare the child for this major step? By simulating aspects of school, starting years before by reading to the child, by talking with the child to enlarge the child's vocabulary, by bringing home learning games with the child. How does the school prepare the child for first grade? By doing things intellectual in kindergarten; by teaching early reading and some math. I do not need to say more. You can see that the ideas are not that much different from the transitions from high school to college.

One hundred years ago, a child entering first grade was not expected to have done much if anything academic before that time. The letters and numbers were introduced at the beginning of the year. Today a child who does not know these things is behind. But there are still people who think that schooling begins when a child goes to school. Sure – formal schooling begins then, but if a child is to be successful, preparation is needed.

The next major transition for mathematics comes in middle school. Whereas in elementary school, everything you needed to learn was discussed in class, in middle school some learning is expected to happen from the homework. Also, reading expectations start appearing for mathematics in the middle school. Symbols for variables, angles and other geometric figures appear. Of necessity, all students must read directions to problems, but in the UCSMP curriculum, we expect students to read the content of their lessons – we aim the writing at them. This begins the process of learning how to learn, perhaps the most important overall consequence schooling can give a person. Lastly, meaningful tests begin to appear for which a student is expected to study.

In high school, usually signaled by the first algebra course or its equivalent, a new set of transitions appears. Whereas in previous years, a bit of content might have been discussed over a few days, and if you were absent or not with it one day, you could get the idea the next, in high school, the material comes quickly. Each day there is something new. A student must learn to focus every day and do homework each night. But another transition is more subtle. The mathematics that is being discussed is not mathematics that you see every day. Through elementary school and most of middle school, the big ideas – fractions, decimals, and percents, the four fundamental operations, the geometric figures, the graphs of a variety of types – all these things can be found easily by going to stores, by looking at familiar objects, or by reading newspapers and magazines. In contrast, algebra is a foreign language. The student now has to work in that language, and to be a good student, become fluent in that language. This requires reading, writing, and speaking, and it is no coincidence that many good students become better by helping or working with their classmates to learn the subject. In that help, they are practicing to make that foreign language their own. They are learning how to learn a foreign language.

I mention these earlier transitions because they are prerequisites to the transitions that a student must make in college, yet we all realize that these earlier transitions are often not made. Furthermore, each transition is greased by a curriculum that attempts to get students to make the transition before it is too late. Thus, for instance, in the UCSMP high school curriculum and in a number of the more progressive high school curricula, there are a large amount of classroom activities to get students to discuss mathematical language with each other and to write cogent arguments, and there are projects and problems that can take a great deal of time and effort.

To repeat: Some students have trouble bridging the gap for college expectations because they have never completely bridged the gap for high school expectations.

Slide 7   2. Linearity in Mathematics

One of the things that is a province of the mathematics curriculum more than the curricula of other subjects is the view that the subject is linear – with a fixed order – arithmetic, then algebra, then functions, then trigonometry, then calculus, then differential equations, and so on. Each step carries with it the potential to psych out a student almost by its mere name. Students are scared that they will not understand the next step in this linear order. Within each of these divisions, there is supposedly a finer order. Within arithmetic: whole numbers, fractions, decimals, percent. Within functions, linear functions, then quadratic functions, then polynomial functions, then exponential functions, then logarithmic functions, etc. Thus an educated adult said to me just this past week, when I was at my 50th high school reunion, I was fine until I got to fractions. Others will say they were fine until algebra, or geometry, or trigonometry. And calculus carries with it the same cachet.

But it is not the case that mathematics is so linear. A child is told his age is 2 1/2 long before the study of fractions. Formulas can be introduced as abbreviations for such things as area = length x width long before algebra. Infinite decimals tend to be discussed in middle school and involve the idea of limits well before calculus. Approximating the area of a region by using finer and finer grids can be done long before calculus. Graphing calculators can show students the graph of a logarithm or trigonometric function even before the student knows how to obtain points on the graph. Introducing the ideas of a topic many times before mastery of the topic is expected generally increases the probability of success when the student gets to the topic. I remember one year when I taught a few days of matrices to a class some years ago – now we teach more, but then we taught only a little – and a student came back to the high school a few years later and told me how happy he was that we studied matrices because he had such a leg up on the other students in his business linear algebra class. We hadn't *studied* matrices; they were only introduced. But that introduction was enough to get the student over any psychological hurdle that might appear when a totally new idea is introduced. These days we have to worry that, when topics are often only discussed if they are on a state exam, that we are not hurting students by not introducing college-level ideas that are too advanced to be tested.

Slide 8 3. Prerequisites for Success in College Mathematics

An assumption of linearity of mathematics is inherent in what the Conley paper says about mathematics core academic knowledge and skills.

Slide 9 (1) *Most important for success in college math is a thorough understanding of the basic concepts, principles, and techniques of algebra. This is different than simply having been exposed to these ideas. Much of the subsequent mathematics they will encounter draws upon or utilizes these principles.* (2) *In addition, having learned these elements of mathematical thinking at a deep level, they understand what it means to understand mathematical concepts deeply and are more likely to do so in subsequent areas of mathematical study.*

Slide 10 I would edit the first sentence to: *Most important for success in college math is a thorough understanding of the basic* skills, *principles,* applications, and representations (graphs) *of algebra*, geometry, statistics, and functions*.* Mathematics literacy of the kind promoted in the Reading in the Disciplines report by Carol Lee and Anika Sprathy comes more from geometry and statistics than from other areas of mathematics. Algebraic literacy comes more from technology such as computer algebra systems, because a student using this technology is forced to interpret mathematics he or she has found but not written. This is what constitutes the kind of understanding that is in the second sentence, and then the second sentence is not needed. Notice that I speak of the understanding of skill – this is an understanding that today is very much helped by the existence of powerful calculators that can do algebra, what we call computer algebra systems.

(3) **Slide 11** *College-ready students possess more than a formulaic understanding of mathematics. They have the ability to apply conceptual understandings in order to extract a problem from a context, use mathematics to solve the problem, and then interpret the solution back into the context.*

This sentence recognizes that algebraic skill does not automatically transfer to the ability to solve problems and I would leave it unchanged. However, the last part of the Conley core violates the spirit of the previous three.

(4) **Slide 12** *They know when and how to estimate to determine the reasonableness of answers and can use a calculator appropriately as a tool, not a crutch.*

The notion that a calculator is a crutch is stated here in a pejorative sense, when the teachers that we work with know that calculators are no more crutches than paper-and-pencil. The good student appropriately uses paper-and-pencil work; the student who does not understand the subject is a slave to answers obtained whether they are found with paper and pencil or with a calculator. But today's calculators are powerful teaching tools, and we have found that the existence of computer algebra systems enhances a student's ability to work with algebra, functions, and calculus, and the existence of corresponding geometry and statistics software does likewise for those subjects. Do not prepare your students for the last century; prepare them for the future. I would modify this statement as follows:

(4) *They are able to move back and forth among the doing of simple mathematics in their heads, the use of paper and pencil technology for short exercises, and the employment of sophisticated calculator and computer technology to handle more complex tasks and explorations; when one method does not solve a problem they can use another method.*

And they need to be prepared for more than calculus in college. Statistics is a first course for as many students as calculus, and the mathematics found in economics, business, and biology is more often than not different from that encountered in calculus preparation. Preparation for college mathematics requires more than preparation for calculus.

Slide 13 4. Some National Data

Now let me turn to the reality of mathematics in schools today and the difficulty of achieving this agenda.

Slide 14 This slide shows percents of 8th graders studying various levels of mathematics curricula. Notice the two right columns. The percent of 8th graders taking something below pre-algebra was 76% 28 years ago and is now 29% according to the most recent National Assessment data.

At the same time, the percent of 8th graders taking algebra or a later course has increased from about 13% to 41%. This constitutes a paradigm shift in our beliefs about algebra in the 8th grade. When only 13% of students took algebra in 8th grade, it was clear that the course was meant for gifted students – the honors track. Only they could take algebra. But when 2/5 of students take the course, then we are talking about a course taken by more people as graduate college.

I am pleased about this increase and I think the work we have been doing at the University of Chicago for the last 25 years has been one of the factors in it.

Slide 15 I will show only one UCSMP slide in this talk. It is this one, a description of the UCSMP courses for grades 6-12. It conveys our belief that students should all be driven on virtually the same roads as they go through their basic mathematical experience in school, but not necessarily at the same ages. All students should be on the same track, but not necessarily in the same place at the same age.

In addition to having algebra as an 8th grade course for students at grade level, there is another aspect of this chart that is uniquely UCSMP. We do not teach algebra in middle schools because we want all these students to take calculus in high school. It takes 5 years, not 4 years for an average student to learn the mathematics from algebra through precalculus. That is one reason why so many students have to take what is called remedial mathematics in college. They have been pushed too quickly through the curriculum.

Slide 16 The increased enrollments in algebra in 8th grade have led to increased enrollments that are at least as dramatic at the high school level. NCES longitudinal transcript studies – a random sample of 10% of the transcripts of H.S. graduates in the country – indicate that from 1982 to 2004, the percent of high school graduates who ended their mathematics experience without ever having studied algebra went down from 25% to 5%. At the same time, the percent of these graduates who had taken a second year of algebra or higher went from 44% to 77%. And it continues today. Over 80% of high school graduates today have credits in both algebra and geometry or their equivalent.

Notice also that the percent of high school graduates who have taken at least precalculus went from 10.7% in 1982 to 33% in 2004, more than tripling.

Slide 17 The vertical lines allow us to take a look at those students who in 2004 completed their high school mathematics with precalculus. These students are in the 67th to 86th percentile of students if we rate students by how much mathematics they have taken when they graduate. Notice that the same percentiles of students in 1982 stopped either with a second year of algebra or with algebra plus trigonometry. These data explain why many high school teachers who have been teaching for many years wonder why their students today are not the same as they were 10 or 20 years ago. They are not from the same population! And the same could be said for the students who complete their mathematics with trigonometry or with any other pre-college course.

When teachers, either at the college level or at the high school level, make the observation that today's students are not as motivated as students were a generation ago, they are almost always insinuating that something is wrong with the curriculum or with the teaching these students have had. I think this slide allows a far more positive interpretation of the same phenomenon: we are teaching more mathematics to more students than we ever did. Most of our students are taking courses their parents never took and, I might say, neither the students nor their parents know why they have to take these courses.

Slide 18 The NCES data show that there has been a huge increase in the numbers of students taking calculus in high school. This is confirmed by Advanced Placement data. From 2000 to 2008 there was a 62% increase in the number of students taking the Calculus AB exam and a doubling of the number of students taking the Calculus BC exam. All this in 8 years.

Slide 19 As a result, as this slide from David Bressoud shows, in 2005-06 more students took calculus in high school than in college, and the AP enrollments suggest that high school calculus enrollments are up 15% since these data were collected.

Slide 20 What is very significant is that the average scores of students on these exams are not going down even though there has been an increase in the population of students in them. Since 2004, despite a 20% increase in the number of students taking the AB exam, about 21% of students have gotten 5's and the percent of scores of 3 or more has remained at about 59%. Despite a 28% increase in the number of students taking the BC test, about 42% of students receive 5's and the percent of scores of 3 or more has hovered around 80%. It means that the quality of results has stayed about the same despite significant increases in the number of students taking the tests.

But, before we get giddy about these data, I'd like to throw out a caution. A couple of years ago I spoke at length with the mathematics department chair at a new small school in Chicago that had been created with the help of funds from the Bill and Melinda Gates Foundation. I asked her what she teaches. She responded that she teaches AP calculus and pre-calculus and 2nd-year algebra. The school has to offer AP calculus because of requirements from the Gates Foundation. I wondered how, in a small school, there would be enough students for an AP class, particularly since in

Chicago very few students take algebra in 8th grade, and probably almost no students in this high school would have had algebra in the 8th grade. She said that the students double up precalculus with calculus. I asked her what scores her students get on the AP exam in AB calculus. She said she has never had a score higher than a 1. This is an example of reform on paper that is not real reform. It is a disservice to these students to make them think they have studied calculus. They are like some of the people who try out for American Idol or So You Think You Can Dance thinking that they have a wonderful singing voice or are a great dancer. They are deluded.

You can see the effects of practices such as those in that Chicago school by noticing that there has been an increase in the number of 1's.

Slide 21 5. An Impossible Dream?

The increased expectations for school mathematics put many schools, like that Chicago school, in a virtually impossible situation. What was high school mathematics now starts for many students before high school, and what was college-level mathematics starts before college. Students are expected to know more mathematics in 4 years between algebra and calculus than ever before. And *all* students are expected to traverse this curriculum, rather than a select gifted group.

Slide 22 Most high schools have to give up one of these goals: either do not have calculus in high school as a goal, or teach a restricted curriculum that aims only towards calculus, or figure that only a small minority of students have the time and can successfully handle the work load of such a demanding curriculum.

Slide 23 The Woodlawn Charter School at the University of Chicago presents an interesting case of a school that has attempted to meet this challenge. It is a 6-12 school and the 6th graders are randomly selected from those who apply. Almost all of them have been in city public schools for the previous grades, and a great number of students are two or more years below national norms.

To catch students up, from 6th grade through 8th grade, the school offers mathematics for two periods a day rather than one. The idea of doubling up classes has been used in many schools for algebra, but the Woodlawn School is the only one I know that doubles up for a number of years. While this practice might seem to be strange, it is not much different from the practice of spending huge amounts of time in early elementary school on reading. The reading and mathematics time increases can be justified because in early elementary school, reading is a sorter, and in middle school, algebra plays the same role. But because the Woodlawn School starts the increased time in 6th grade, the school can offer algebra in 8th grade to a great number of students. But, even more important, the extra time devoted to algebra ensures that the vast majority of students get a strong foundation for future high school courses. These courses will give a broad mathematics education, including a good deal of statistics and applied mathematics as well as the traditional preparation for calculus. The first 6th grade class is only now in 9th grade, so we do not know what will happen, but I think the school's mantra gets it right. The mission of the school is not to give them an education that will enable children to get into college, it is to give them an education that will prepare for success in four-year colleges. To place this in our context here, the idea that readiness for college begins at the 9th grade has been expanded by operating on the assumption that readiness for high school begins no later than 6th grade.

Can this idea be replicated on a wide scale? The Woodlawn School is open from 7AM to 7PM. It is backed by grants from a large number of organizations and the support of the University of Chicago. That alone is enough to indicate the magnitude of the problem. But, in addition, there are aspects of this problem that are not academic, but economic. In the U.S., a large number of high school students work significant hours each week. They do so during high school and they do so during college. The learning of any subject takes time, but a subject like mathematics, where practice is necessary for success, requires more time. Surveys over decades have shown that high school students spend more time on mathematics than other subjects, yet the average grades given by mathematics teachers are lower than those in the other disciplines. There is no secret as to why children from homes in which the parents are able to say “You do not have to work; school is your job” perform better than others. The overall solution to bridging the gap between proficiency and college readiness involves subject-matter and broader study habits, but it also has a major economic component.

Thank you!